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Teaching Children How to Use Language to Solve Maths Problems

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It is often claimed that working and talking with partners while carrying out maths activities is beneficial to students' learning and the development of their mathematical understanding. However, observational research has shown that primary school children often do not work productively in group-based classroom activities, with the implication that they lack the necessary skills to manage their joint activity. The research we describe investigated these issues and also explored the role of the teacher in guiding the development of children's skills in using language as a tool for reasoning. It involved an interventional teaching programme called *Thinking Together,* designed to enable children to talk and reason together effectively. The results obtained indicate that children can be enabled to use talk more effectively as a tool for reasoning; and that talk-based group activities can help the development of individuals' mathematical reasoning, understanding and problem-solving. The results also encourage the view that the teacher of mathematics can play an important role in the development of children's awareness and use of language as a tool for reasoning. We suggest some ways that this role can be carried out most effectively.

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Keywords: collaborative learning, mathematics education, classroom dialogue, primary schools, teaching

Introduction

In recent years, researchers have paid increasing attention to the role of language and social interaction in the learning and pursuit of mathematics (e.g. Barwell *et al.*, 2005; Forman & van Oers, 1998; Hoyles & Forman, 1995; Monaghan, 1999; Sfard, 2000; Sfard & Kieran, 2001). This interest relates to the function of language in both teacher–student encounters and in peer group activities. For example, Yackel *et al.* (1991) carried out a study in which all maths instruction in a primary classroom was replaced by problem-solving activities in small groups. While emphasising the value of the teacher's guiding role, they found that the group activities offered valuable opportunities for children to construct solutions for themselves through talk which would not be found in whole-class instruction. Focusing more on the teacher's role in leading classroom

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conversation, Strom *et al.* (2001) analysed the ways a teacher used talk to guide the development of children's mathematical argumentation. Taking a rather different perspective, sociological researchers such as Dowling (1998) have considered how the discourse of mathematics education is constructed through pedagogic practices, and how this affects the accessibility of the subject for some children. Others, such as Barwell (2005), have argued that the tendency amongst policymakers and maths educators to stress the distinction between the precise, subject language of mathematics and more informal talk can hinder the process of inducting children into mathematical practices.

There is a well-established field of research on teacher–student interactions in classrooms, some of which has been directly concerned with the effectiveness of teachers' discourse strategies for assisting students' learning and development (as reviewed, for example, in Edwards & Westgate, 1994; Mercer, 1995). The study of group activities in the classroom, from the point of view of their value for assisting learning, has also become well established (see e.g. Barnes & Todd, 1977, 1995; Bennett & Cass, 1989; Blatchford & Kutnick, 2003; and with special relevance to mathematics education, Hoyles & Forman, 1995).

For researchers who take a neo-Vygotskian, sociocultural perspective, interest in language is related to its functions in the learning and cognitive development of individuals. Vygotsky (1978) argued for the importance of language as both a psychological and cultural tool. He also claimed that social involvement in problem-solving activities was a crucial factor for individual development. As he put it, intermental (social) activity – typically mediated through language – can promote intramental (individual) intellectual development. This claim, having an obvious plausibility, has been widely accepted. However, other than our own earlier findings, any empirical evidence offered for its validity has been, at best, indirect. Our earlier research showed that the induction of children into an explicit, collaborative style of reasoning which (following Barnes and Todd, 1977, 1995) we call *Exploratory Talk* led to gains in children's individual scores on the Raven's Progressive Matrices test of non-verbal reasoning (Mercer *et al.*, 1999). These findings, first demonstrated for children in Year 5 in British primary schools, were subsequently replicated in other year groups and in primary schools in Mexico (Rojas-Drummond *et al.*, 2001). We have also demonstrated the positive influence of Exploratory Talk on children's understanding of science and their attainment in formal science assessments (Mercer *et al.*, 2004). An additional important aspect of that research has been to highlight the potential significance of the role of the teacher as a 'discourse guide', someone who scaffolds the development of children's effective use of language for reasoning through instruction, modelling and the strategic design and provision of group-based activities for children (Mercer, 1995; Rojas-Drummond & Mercer, 2004). As we will go on to show, we can now offer evidence of similar effects of teacher guidance and involvement in structured discussion for the study of mathematics.

The Aims and Focus of the Research

With relevance to the topic of this paper, the research was designed to test the following hypotheses:

(1) That providing children with guidance and practice in using language for reasoning will enable them to use language more effectively as a tool for working on maths problems together.
(2) That improving the quality of children's use of language for reasoning together will improve their individual learning and understanding of mathematics.
(3) That the teacher is an important model and guide for pupils' use of language for reasoning.

The study was based in primary schools in South East England and involved the implementation of a teaching programme for children in Year 5 (ages 9–10). As we will explain, this teaching intervention was designed to develop children's use of language as a tool for reasoning together and to enable them to use the discussion skills they developed in carrying out mathematical activities. The basic design of the activities in this Thinking Together programme was based on the outcomes of a series of earlier, related projects into the relationship between the language of teaching and learning and the development of children's knowledge and understanding (as described in Dawes, 2005; Mercer, 1995, 2000; Mercer *et al.*, 2004; Wegerif *et al.*, 1999; Wegerif, 2005; Wegerif & Dawes, 2004).

Classroom Dialogue in the Teaching and Learning of Maths

As mentioned in the introduction, there are two main kinds of interaction in which spoken language can be related to the learning of maths in schools. The first is *teacher-led interaction with pupils*. A sociocultural account of cognitive development emphasises the guiding role of more knowledgeable members of communities in the development of children's knowledge and understanding, and this kind of interaction can be important for their induction into the discourses associated with particular knowledge domains. Elsewhere, we have described this as 'the guided construction of knowledge' (Mercer, 1995). Also very relevant is the concept of 'dialogic teaching', as recently elaborated by Alexander (2004). His cross-cultural research (Alexander, 2000) has revealed the very different ways that teachers can make use of classroom dialogue when interacting with children. The variation he describes is not revealed by superficial comparisons of the extent to which teachers use questions or other kinds of verbal acts: rather, it concerns more subtle aspects of interaction such as the extent to which teachers elicit children's own ideas about the work they are engaged in, make clear to them the nature and purposes of tasks, encourage them to discuss errors and misunderstandings and engage them in extended sequences of dialogue about such matters. In his 2004 publication, written mainly for a practitioner audience, Alexander suggests that dialogic teaching is indicated by certain features of classroom interaction such as:

- questions are structured so as to provoke thoughtful answers [. . .];
- answers provoke further questions and are seen as the building blocks of dialogue rather than its terminal point;
- individual teacher–pupil and pupil–pupil exchanges are chained into coherent lines of enquiry rather than left stranded and disconnected. (Alexander, 2004: 32)

Alexander's findings resonate with our own evaluations of teachers' interactional strategies (Rojas-Drummond & Mercer, 2004) and that of researchers in science education (Mortimer & Scott, 2003), with our own research showing that the use of more 'dialogic' strategies achieved better learning outcomes for children. Moreover, our research has shown that teachers can act as important models for children's own use of language for constructing knowledge. For example, a teacher asking children *why* they have gone about an activity in a particular way can be very useful for revealing their perspective on the task to the teacher and for stimulating their own reflective enquiries.

The second context is that of *peer group interaction*. Working in pairs or groups, children are involved in interactions which are more 'symmetrical' than those of teacher–pupil discourse and so have different kinds of opportunities for developing reasoned arguments, describing observed events, etc. In maths education, such collaboration can be focused on solving problems or practical investigations, which also have potential value for helping children to relate their developing understanding of mathematical ideas to the everyday world. Computer-based activities can offer special opportunities for maths investigations, such as games which have mathematical principles and problems embedded in them. However, observational research in British primary schools has shown that the talk which takes place when children are asked to work together is often uncooperative, off-task, inequitable and ultimately unproductive (Bennett & Cass, 1989; Galton & Willamson, 1992; Wegerif & Scrimshaw, 1997).

A possible explanation for the doubtful quality of much collaborative talk is that children do not bring to this task a clear conception of what they are expected to do, or what would constitute a good, effective discussion. This is not surprising, as many children may rarely encounter examples of such discussion in their lives out of school – and research has shown that teachers rarely make their own expectations or criteria for effective discussion explicit to children (Mercer, 1995). Children are rarely offered guidance or training in how to communicate effectively in groups. Even when the aim of talk is made explicit – 'Talk together to decide'; 'Discuss this in your groups' – there may be no real understanding of *how to* talk together or for what purpose. Children cannot be expected to bring to a task a well-developed capacity for reasoned dialogue. This is especially true for the kinds of skills which are important for learning and practising mathematics, such as constructing reasoned arguments and critically examining competing explanations (Strom *et al.*, 2001).

On this basis, there are good reasons to expect that children studying maths would benefit from teacher guidance in two main ways. First and most obviously, they need to be helped to gain relevant knowledge of mathematical operations, procedures, terms and concepts. Teachers commonly expect to provide this kind of guidance. Secondly, they need to be helped to learn how to use language to work effectively together: to jointly enquire, reason, and consider information, to share and negotiate their ideas, and to make joint decisions. This kind of guidance is not usually offered. One of the practical aims of the Thinking Together research has been to enable teachers to integrate these two kinds of guidance.

Method

The research involved the design, implementation and evaluation of an experimental programme for developing children's use of talk as a tool for reasoning and carrying out collaborative activity, and then directing their use of language for the study of mathematics and science. We will describe the design of the study, the intervention programme, how data was gathered and the methods used to analyse it (in relation to the maths strand only: the science strand of this work has been reported in Mercer *et al.*, 2004).

The design of the study: Target and control classes

The research involved, in total, 406 children and 14 teachers in schools in Milton Keynes and was carried out in 2002–4. The effects of the intervention programme on children's talk, reasoning and learning were studied through observation and formal assessment in experimental 'target' classes, with pre- and post-intervention comparisons being made with children in matched control classes in other local schools with similar catchments. No special criteria were used to select teachers in target or control schools, other than their enthusiastic willingness to participate in research on children's talk. Seven teachers and their target classes of children in Year 5 (aged 9–10) participated actively by following the Thinking Together intervention programme. There were 196 children in the target classes at the beginning of the study, but with children arriving and departing from classes during the intervention period, maths-related data on just 109 of the original children was available at the end of the study. (Milton Keynes has a high rate of population movement, within the city as well as into and out of it.) A matched set of 'control' classes in seven other similar local schools were identified. These consisted of 210 children at the beginning of the study, with 121 of those still being in the control classes for the last data collection. Control classes did not participate in the Thinking Together programme, but followed the same prescribed National Curriculum for Year 5. They also had access to the same software provided by the National Numeracy Strategy and used in our teaching programme (as described below), but were not required to use it as a basis for regular group activities.

The intervention programme

As mentioned previously, our earlier research had shown that language skills associated with improved reasoning can be effectively taught and learned. That research involved the creation and evaluation of talk-based classroom activities. An intervention programme was designed for the present study which built directly upon that earlier research. A key feature of the Thinking Together programme in recent projects had been the systematic integration of teacher-led interaction and group-based interaction. The main aims of the specific version of the programme in this study were:

- to raise children's awareness of the use of spoken language as a means for thinking together;
- to enable children to develop their abilities to use language as a tool for thinking, both collectively and alone;

- to enable children apply the tool of language effectively to their study of the science and maths curriculum.

More specifically, the programme was intended to ensure that children became able to carry out the kind of discussion which, following Barnes & Todd (1977, 1995), we call *Exploratory Talk*. This is talk in which:

- all relevant information is shared;
- all members of the group are invited to contribute to the discussion;
- opinions and ideas are respected and considered;
- everyone is asked to make their reasons clear;
- challenges and alternatives are made explicit and are negotiated;
- the group seeks to reach agreement before taking a decision or acting.

Teacher induction

All the teachers of the target classes were given an initial training session about Thinking Together. In this initial session, the underlying principles and concepts of the Thinking Together approach were explained and discussed. Teachers also engaged in group problem-solving activities and then were asked to reflect on the ways that they did so. Through the use of videos and transcripts of classroom talk, they jointly engaged in analysing examples of the talk of children working in groups (for example, identifying features of Exploratory Talk) taken from earlier phases of our research. They were also asked to reflect upon and discuss together the ways that they explained, guided and modelled talk for children, and how they set up children's participation in working groups. They were offered some specific strategies for doing so (based on earlier research, as described in Mercer, 1995, 2000; Rojas-Drummond & Mercer, 2004). The initial half-day training 'workshop' was followed up with regular 'twilight' meetings and discussions in their schools during the researchers' visits. Teachers of control classes did not receive training but were told that the results of the research would be made available to them on its completion. We have no reason to believe there was any relevant exchange of ideas between target and control schools during the period of study. The complete set of the lessons (including those concerned with science, which are not discussed here) were implemented in the target schools over the Autumn and Spring school terms, with the total duration of the intervention (i.e. from the pre-testing to the post-testing) being 23 weeks.

Lessons and activities

Each 'target' teacher was provided with 12 detailed lesson plans. These lessons all had the same format, which is one commonly used for lessons in English primary schools. Each began with a teacher-led whole-class introduction, followed by a group discussion activity and then a final, whole-class plenary session in which ideas were shared and there was some reflection on the outcomes of the activities of the lesson as a whole. The aims of each lesson included the development of talk skills such as critical questioning, sharing information and negotiating a decision. These were closely related to the speaking and listening component of the National Curriculum for English.

Activities in the first five lessons were largely concerned with raising children's

> We will:
> Discuss things and ask questions
> Include everyone's ideas
> Ask what people think and what their reasons are
> Cooperate to work together
> Listen to each other
> Make an agreement before deciding

Figure 1 Class 5's ground rules for talk

awareness of how talk could be used for working together and establishing in each class a set of 'ground rules' for discussion which would generate talk of an 'exploratory' kind. An example of a set of rules established by one teacher with her class is in Figure 1.

The first five lessons were completed in the target schools by the end of the first term (which also saw the completion of the pre-intervention observations and assessment in both target and control schools). The seven following lessons were designed to enable children to apply their developing discussion skills to the study of the prescribed Year 5 maths and science curriculum. Each lesson applied a specific talk skill and targeted a specific concept in maths (or science). In these lessons, teachers also regularly reinforced the importance of effective discussion in group activity by invoking the ground rules which had been established in the first five lessons. This is illustrated by Transcript 1: *Reviewing the Ground Rules*, below, which comes from the introductory part of a mathematics lesson in a target school. As preparation for a group-based activity, the teacher is eliciting from the members of the class the ground rules they had earlier established. As she questions the class, the teacher writes the rules they offer in speech bubbles on the whiteboard.

[*In all the transcripts which follow, standard punctuation has been used to represent the grammatical organisation of the speech as interpreted by the researchers. Words spoken emphatically are underlined. Non-verbal actions and other information judged relevant to the interpretation of the dialogue are described in italics.*]

Transcript 1: Reviewing the Ground Rules

Teacher: How are your Thinking Together skills going to help you with that? Why do you need to do that in your Thinking Together group? Kelly?
Kelly: We need to talk about it.
Teacher: Why do you think we need to talk about it?
Kelly: To get more ideas.
Teacher: Excellent. If we talk about it we might have a few more ideas than just working on our own.

Paula: And because you can't just think that it's the answer when somebody else has got another idea – you have to check with the group – see what they think.

Teacher: Excellent. OK. So if I walk around the classroom while everybody is talking together in their groups I wonder what kind of things might I hear people saying?

Asif: 'What do you think?'

Teacher: That's a good one. Why is that an important question Carl?

Carl: Because you ask someone else their opinion.

Sarah: 'I think this because.'

Teacher: Why did you add 'because' to the end of that sentence?

Girl: Because then they know why you made that remark.

Teacher: Well done. Brilliant. You need to explain so that everybody understands what you think.

We can see that the teacher's questioning elicits reasons for using a collaborative approach and ways it can be enacted through talk. The children's responses indicate an awareness of what they might achieve through thinking together. Thus, the culture of collaboration is constructed through the teacher's line of questioning.

The seven lessons of the programme which were focused on maths and science included some computer-based activities (the use of ICT in the Thinking Together research is discussed in Wegerif & Dawes, 2004). For the maths lessons, activities included some taken from the mathematical software provided to British teachers by the government's National Numeracy Strategy (DfES, 2001) and involved children in reasoning about number relations. One activity, which will figure in later examples of classroom talk, used a piece of software called *Function Machine* in which the children were asked to consider what operation could be carried out on one number in order to end up with another (for example, 'divide it by two and add 2') . Children worked in groups of three to discuss their ideas about how to solve the problems. The content and level of all these activities was closely related to the mathematics curriculum and Numeracy Strategy for Year 5 in English schools, as prescribed for all the schools involved in our study.

Researchers tried to ensure that all the lessons in the intervention programme were carried out in each target school and this was achieved for the initial five lessons. For a variety of practical reasons some teachers were unable to fit all the later lessons into their class's activities, but all carried out at least three of the seven; and in any case we observed that most target class teachers also used the Thinking Together methods in their regular lessons in maths and other subjects.

Data collection

The data gathered included:

(1) pre- and post-intervention video recordings of a 'focal group' in each target class carrying out computer-based activities;

(2) video recordings of other groups of children in the same classes engaged in joint activities during Thinking Together lessons;
(3) video recordings of teacher-led whole class sessions during Thinking Together lessons;
(4) post-intervention audio recordings of interviews with teachers and children;
(5) pre- and post-intervention tests of children's knowledge and understanding in maths and science.

The tests used as pre- and post-intervention measures of learning in maths were based on the so-called 'optional Standard Attainment Tasks' (SATs) for Key Stage 2 which are provided to all schools in England, Wales and Northern Ireland by the Qualifications and Curriculum Authority. Test items (7 per test) focused on data handling, properties of numbers and number sequences, addition and shape, all of which were taught to the children in the control and target schools as part of the statutory National Curriculum for Year 5.

Data for (1) to (5) above was gathered in all target schools, with pre- and post-intervention data for only (5) also gathered in the control schools. Interviews and video recorded interactions were transcribed. The data we gathered was intended to inform us about:

(1) the teacher's role in guiding and modelling children's use of language and their problem-solving in maths (and the effects of our intervention on this role);
(2) the nature, content and outcomes of children's group discussions;
(3) the outcomes of our intervention on the quality of children's discussion;
(4) the outcomes of our intervention on children's understanding of maths.

To study the teacher's role in guiding children's use of talk and learning, we video recorded all of the target class teachers carrying out at least two of the prescribed lessons. Other lessons were also observed, but not recorded, by researchers. In order to study the effects of the Thinking Together programme on curriculum-related learning, we compared target and control children's understanding of National Curriculum maths topics that they had studied during the period when the intervention took place. We describe this assessment in more detail below.

Methods of analysis

Evaluating children's knowledge and understanding of the maths curriculum

As mentioned above, improvements in children's mathematical knowledge and understanding were assessed at the end of the intervention programme using problems taken from the optional SAT tests for Key Stage 2, covering topics studied by children in both target and control schools. Additional evidence came from teachers' informal assessments of children's progress made over the same period.

Analysis of teachers' interactions with children

As explained above, we were interested in how teachers can act as models and guides for children's use of language and our analysis sought to identify ways in which this could be seen to take place. The particular types of use on which we focused relate to the aim of the intervention in encouraging the use of Exploratory Talk. We therefore looked particularly at the extent to which teachers:

(1) used 'why' questions, in which they sought children's reasons for holding an opinion, or for having carried out a particular operation;
(2) used 'reasoning words' such as 'if', 'because', 'so';
(3) offered reasons of their own to back up statements or proposals;
(4) checked that everyone who had a relevant idea had been heard;
(5) sought agreement amongst the class at the end of a debate.

Analysis of children's talk in groups

Video recorded data was also used to investigate changes in the quality of children's talk and collective reasoning. The methodology for making this kind of comparison, as described in more detail in Wegerif and Mercer (1997) and Mercer (2004), combines a detailed qualitative analysis of language used by each group of children in specific episodes of joint activity with a quantitative computer-based analysis of the whole corpus of recorded group talk. These methods were used to:

(1) examine how children in all the classes used talk to solve problems;
(2) look for changes in the pre-intervention and post-intervention talk of children in target classes.

We thus sought evidence of changes in the quality of children's talk (pre/post-intervention) which related to our hypotheses. We also sought to determine the extent to which the intervention had been more or less successful in the different target schools. For reasons of space, in this paper we will focus only on some aspects of our analysis, the first of which is what it revealed about changes in the quality of talk and reasoning in groups in the target classes. This essentially involved gauging the extent to which the discussions of children in the target classes came to resemble Exploratory Talk. However, the analytic process has more subtle features than this may seem to suggest. We were not interested in simply identifying stretches of talk as 'exploratory' and coding them accordingly. Indeed, our view is that the nature of language – in which any one grammatical form can be used to fulfil a range of pragmatic functions – renders any coding scheme of dubious value if used separately from a more contextually sensitive, ethnographic type of analysis. Our aim has been to consider the extent to which children are using language appropriately and effectively as a tool for thinking together. In making that analysis, the definition of Exploratory Talk serves as an 'ideal type' – a typification of reasoning embodied in talk. The features of Exploratory Talk, as described earlier, are therefore used as a point of reference for the consideration of the quality of the talk of each group. The nature of this qualitative assessment in practice is illustrated by our analytic comments following the presentation of the two examples of children's talk, Transcripts 4 and 5, later in this paper. More details of this process can also be found in Mercer (2004). It is also worth noting that the focus in this paper is only on children's use of talk for engaging in mathematical problem-solving, but this does not mean that our analysis is insensitive to other cognitive and social functions of talk in groups. We are well aware that, while working in classroom groups, children use talk to do much more than engage in curriculum tasks: they form relationships, develop social identities, and pursue 'off-task' activities which may be more important to them than the tasks in which they officially engaged – and, as Wegerif (2005) has argued, may be essential to the process of establishing good relationships so that effective 'on-task' activities result.

Results

We have elsewhere described the outcomes of the Thinking Together intervention on children's talk in groups, using pre- and post-intervention comparisons between children in target and control classes (e.g. Mercer, 2000; Mercer *et al.*, 1999). Essentially, both qualitative and quantitative analyses of children's talk in groups showed significant increases in discussion which resembles Exploratory Talk amongst target classes. We have also reported on the effects of the intervention on children's performance on a test of non-verbal reasoning, and on their attainment in science (Mercer *et al.*, 2004). We will not duplicate those earlier publications by elaborating those findings here. Instead, we will begin by reporting the results of the intervention on measures of children's understanding of mathematics (which have not been reported elsewhere) and then consider in more detail the role of the teacher in making this kind of intervention effective.

Evaluating children's knowledge and understanding of the mathematics curriculum

The results of the assessment based on maths SATs questions are shown in Table 1.

The results show that the children in the target classes improved their assessed attainment in maths significantly more than those in the control classes. These results, together with more informal evidence from teacher assessments, thus support the view that the intervention was effective in improving children's study of the maths curriculum. In their recorded comments to the research team, the teachers in target schools also reported that the children generally engaged more collaboratively, enthusiastically and productively in the group activities. Taken with our findings related to the study of science and the development of

Table 1 Results on the SATs maths items

	Numbers	*Pre-intervention: mean scores*	*Post-intervention: mean scores*
Target classes	109	2.43	5.53
SD		1.858	2.452
Control classes	121	2.39	4.20
SD		2.002	2.233
Effect size	0.59		

$F(1,228) = 28.394$; two-tailed $p = 0.000$

general reasoning skills, as reported elsewhere (Mercer *et al.*, 1999, 2004; Rojas-Drummond & Mercer, 2004), these results add to a substantial body of evidence that teaching children how to use language as an effective tool for collaborative activity has a significant and beneficial effect on their educational participation and achievement.

The teacher's role as 'discourse guide'

As explained above, the Thinking Together intervention involved teachers in workshops in which they were asked to reflect on the ways they explained, guided and modelled children's participation in working groups, and to try using some specific strategies for doing so. One of our interests was in the extent to which this in-service aspect of the intervention was effective. We wished to see how teachers implemented the Thinking Together intervention through their talk with children during the plenary sessions of lessons, and in their interactions with children while engaged in group activities. We will focus here on the extent to which teachers modelled Exploratory Talk when interacting with the children, as it is the aspect most directly involved with the teacher's role in shaping children's use of language for solving mathematical problems. To do so, we will make some comparisons between the ways two teachers in our target schools interacted with children during a Thinking Together lesson.

In Transcripts 2 and 3 below, we show how the two teachers involved in the intervention in target schools used part of the initial whole-class session of Thinking Together Lesson 8 to introduce a maths activity using the software called *Function Machine*, in which the children are asked to consider what operation might have been carried out on one number in order to end up with another. As well as deciding on the operation, the groups have to come up with a strategy for discovering it and for testing their ideas about it.

Transcript 2: Teacher A

Teacher: OK, I'm going to make it like a bit of a quiz – something for you to think about in your groups. If you hit 'Random' the machine is going to decide on a rule for itself. Here's the machine. This is the bit where you put the numbers into the machine. The machine does some work on them and it has an output box where the numbers go to once it's done its work on them, OK? So, to put a number in you need the cursor in the input box then put a number in so. [*A child keys in '4'*] Four, thank you Amos, four it is. Now all we need to do is activate the machine. This thing lights up when you hover over it so hover over that, activate that and it has turned it into minus one. Now your job as a group is to try and think what might the machine be doing. Discuss that in your groups and when you come up with an idea, test what you think by putting some more numbers in. Has anyone got any ideas as to what the rule might be for an example? Alan? Alan says it might be 'take away five', four take away 5 would be minus one. What does anybody else think? Well we'll try that. So we need to clear it and put some more numbers in to test it. So he says take away five. Let's put another number in so we can test it by taking away five. Two? Right, if we put two in and it is take away five, what should it be? Come in Laura, come and sit down. Minus three? Minus three so if our rule is right, and we activate it, it will come up with minus three. That's what you are trying to do, see what the rule is, then test it with more numbers [*Activates software*] Oh! Minus five. Oh dear. So what would you do now? What would you have to do now in your group? You'd have to think about it again and see if you can think of another rule it might be.

Alan: It could be minus two.

Teacher: Um – I don't think so. When you have eventually worked out what it is this box down here reveals the programme. This is quite a hard one really. This one says 'Double the input and subtract nine'. But a lot of them are a bit more one-stage operations, like add 4, multiply by three, divide by two something like that. So if you get mega-mega stuck and you try it several times and you can't work it out you can have a look at it. And then think of a number you can put in and see if you can say what will come out. So we know this doubles the input and subtracts nine, so think of a number we could put in and what would come out if it's doubling the input and subtracting nine. OK? Mary?

Mary: Eighteen.

Teacher: Eighteen. So if we put eighteen in and double the input what are two eighteens – thirty-six? And then subtract nine – what's thirty-six take away nine? [*A child responds – inaudible*] Twenty seven? – Yes twenty-seven. OK let's try it then – so if we get rid of that – I think this is going to work – put eighteen in, activate that – yes twenty-seven. So once you've done that you can start all over again with a different thing. You press clear to clear it all then select 'Random' from down in this bottom corner now the machine has got a new rule in it – shall we try this one? Give me a number . . . thirty-six. Oh I forgot the cursor that's why it wouldn't go in – we need the cursor – remember that. Now – thirty-six went in, Activate . . . thirteen! Thirty-six went in, thirteen came out.

Elenor: Take away three and take away twenty.

Teacher: What might it have done? You'd be in your groups now saying what might it have done. One of you would say something and then someone would say something else, they you'd discuss it and try it.

Elenor: [Teacher's name] it's twenty three.

Teacher*:* Well that's where I'm going to leave you to try that.

Transcript 3: Teacher B

Teacher: OK. I'm going to put a number in (*looks at class quizzically*).

Louis: One thousand.

Teacher: OK Louis immediately said one thousand. Is that a good number to put in?

Child: No.

Teacher: You are shaking your head. Why do you think it is not? Shall we come back to you? You've got an idea but you can't explain it? OK Louis had one thousand. Anybody think yes or no to that idea? David.
David: Start off with an easier number.
Teacher: Start off with an easier number. By an easier number what kind of number do you mean?
David: Um. Something like, lower, five.
Teacher: Fine. A smaller number, a lower number, yes. Louis can you see that point of view?
Louis: Yes.
Teacher: If we put in a thousand we could end up with a huge number. If we put in five do you think it will be easier to work out what the machine has done?
Class: Yes.
Teacher: Everyone agree?
Class: Yes.
Teacher: OK, I'm going to type in five.

Comment

The first teacher essentially engages in a monologue, which runs through the procedures which the children will have to follow. The information provided is very relevant, but the event is not very interactive. The children are asked for some suggestions, but Teacher A provides few opportunities for them to do so. Questions are not used to elicit reasons, or to explore children's understanding: they are used mainly to elicit arbitrary numbers for putting into the machine. Even when the teacher appears to ask for their opinions, a response slot is not provided for them to do so; the teacher answers the question posed. (For example: 'So what would you do now? What would you have to do now in your group? You'd have to think about it again and see if you can think of another rule it might be.' And 'You'd be in your groups now saying what might it have done. One of you would say something and then someone would say something else, they you'd discuss it and try it.') No clear feedback is provided to the responses by Alan and Elenor; Teacher A does not seek the reasoning behind them, or use techniques such as reformulations to ensure that children's contributions are represented publicly and clearly in the dialogue. This teacher does not model features of Exploratory Talk.

In contrast, the second teacher embodies some of the ground rules for Exploratory Talk in whole-class dialogue. Like the first teacher, Teacher B shares relevant information with the class about the nature of the number which is to be put into the input box of the function machine. But this teacher also initiates discussion about the number by questioning the first suggestion made by a pupil, and follows this with requests for reasons for opinions and assertions. The language used in this whole-class session shows some of the common features of teacher talk: lots of questions, repetitions, and reformulations. However, Teacher B uses these not simply to quiz children about their factual knowledge, or to correct their factual knowledge, but to engage them with the problem and ensure that their views are represented in the dialogue. The teacher's contributions include 'reasoning words' such as 'what', 'how', 'if' and 'why' as the children are led through the activity. The teacher accepts and discusses the challenges made to Louis's suggestion, while respecting the contribution he made in initiating the discussion. The children are given a demonstration of how to consider the validity of alternative suggestions. The teacher invites children to speak so that as many people as possible are able to join in the discussion – and finally ensures that an agreement is sought and reached. In this way, through careful modelling of the ground rules for talk, the teacher is demonstrating to the children how effective collaboration can be as an integral element of intellectual activity. None of the children makes an extended contribution to the dialogue, so it may be that this interaction does not serve as a very good example of what Alexander (2004) calls 'dialogic teaching'. But the children are engaged in the discussion, their points of view are sought, they have some influence on the discussion and the actions that are taken. And by being engaged in a dialogue in which Exploratory Talk is modelled, they are being prepared to use it when they continue the activity in small group discussion.

Our classroom recordings show that Teacher A rarely modelled Exploratory Talk during the introductions to the lesson. Although the class had agreed a set of ground rules for talk in the early part of the project, no time was spent recalling these with the class; the children were just told that they would be working in their 'Thinking Together' groups to carry out the maths activities. In addition, little time was spent in working with the class on an activity during the introduction to the lesson; the possibility of developing group strategies to solve problems was not discussed.

In contrast, our observations show that the Teacher B commonly modelled Exploratory Talk with the class during whole class introductions. In all parts of the introduction, the teacher would initiate discussion with the class and often gave opportunities for children to talk together before making contributions to the whole class discussion. Teacher B made it clear that reasons should be given to support suggestions, that ideas could be challenged and that alternatives would be considered before attempting to reach a class agreement. Time was also spent in shared recall of the class's 'ground rules' for talk and in discussing why each of these was important. Examples of the kinds of things that might be heard if this kind of talk was going on were elicited from the class and recorded on the whiteboard. Finally, there was some discussion of the need to devise a strategy to solve the mathematical problem. Teacher B's engagement with the children was thus more 'dialogic' in both its structure and content.

Analysis of children's talk and collaborative activity

We have selected two examples of children's discussion in groups to illustrate some kinds of variations in the talk we observed which can be related to the impact of the Thinking Together intervention on joint problem solving in maths. They also come from lessons using *Function Machine* and are taken, respectively, from the classrooms of the teachers who feature in Transcripts 2 and 3 above. The children are expected to talk together to agree on a number to enter, then they are asked to consider the output number that is generated. This information is used to form a hypothesis about the function which has been applied to the original number, and this is then tested. The lesson plan provided for the teachers stressed that one aim of the group work should be for the children to try to devise a strategy for identifying the function involved.

Transcript 4: Group A: Sylvia, Alan and Sabena

Alan: . . . 39 add 5. Half the input is 30, half of 9 is . . .
Sylvia: We can't do this number 'cos we can't do decimals. Let's start again. *(She enters a number)*
Alan: OK, 30. That's your turn.
Sylvia: Twenty-eight. I've got a rule right – if you halve that it's 30 then you take away 10, and then from the 30 take away . . .
Alan: I've got an idea. That's 14, then you're adding 2.
Sylvia: I know – I've got it half the input . . .
Alan: It's my turn.
Sylvia: No – you don't know what to do – I <u>know.</u>
Alan: Yeah but it's my turn.
Sylvia: Wait . . .
Alan: No. Me and Sabena should have two turns then.
Sylvia: No, but wait a minute. I didn't have a turn before. I didn't have a turn. 33 and then add 5. *(Presses key to reveal answer.)*
Alan: My turn.
Sylvia: I've got an idea, I've got an idea.
Alan: You're always having a turn.
Sylvia: Yeah, but I'm faster than you and you can't do anything.
Alan: No.
Sylvia: Shut up *(inaudible)*
Alan: Shall we go with the first number I had?
Sylvia: Uh, go with a number that's very easy– like, uh, 15.
Alan: One.

Transcript 5: Group B: Kylie, Rebecca, Maya and Tony

Kylie: 3! I think it's take away . . .
Rebecca: What do you think? I think it's take away 3.
Maya: Half
Kylie: Half the number. I think it's half the number.
Maya: Me too. Maybe . . .
Tony: Yeah.
Maya: Let's try number 4.
Kylie: 4?
Maya: Yeah – should be 2
Rebecca: Click on there. Click! *(Indicates to Maya where to click.)*
Kylie: Stop arguing. We're being recorded and we've got a microphone. We didn't agree on 4 did we?
Others: Yeah.
(4 entered and 2 appears in output)
Kylie: *(To Tony)* So what do you think?
Tony: I think you have to add on two more.

Kylie: No 'cos, I think like Rebecca, I think it's halving because we had 6, and it ended up 3. Now we've got 4 and it ended up in 2. Do you think half the number or subtract? Do you want to check? Do you want the reveal thing?
Tony: No, I think it's what Maya said.
Rebecca: What did you say?
Maya: I said try 4 and it would come out half.
Kylie: Tony, do you want to try a different number, try once more?
Rebecca: Let's see if we put in an odd number and see what happens.
Kylie: Yeah an odd number. *(Short interruption while they adjust the seating to make sure that Tony has enough room and remains included in the group.)* Do you want to all try 5?
Rebecca: Try 5 yeah?
Kylie: Do we all agree?
Tony: Yeah.
Maya: Nought . . . 2.5
Kylie: We thought it was half the number. *(To teacher, who has joined the group)*.
Rebecca: Half the number.

The children in Transcript 4 are not collaborating effectively, and this was typical of their interaction throughout the activity. They do not attend to each other's suggestions and no agreement is sought about how to proceed (e.g. about abandoning the first number). Children act individually: Sylvia's assertion that 'we can't do decimals' remains unchallenged and unsupported; she enters a new number without any consultation with the rest of the group. Although Alan and Sylvia both have ideas about the function in the next part of the transcript, they voice their thoughts in parallel rather than interacting with each other's assertions. After this the talk degenerates into a disputational exchange about turn-taking. The third child in the group (Sabena) is ignored, says nothing and is only referred to by Alan to back up his claim that Sylvia is having more than her fair share of turns. Their obsession with turn-taking shows that the group does not have an effective, shared set of ground rules for productive interaction.

Transcript 5, in contrast, is illustrative of a more collaborative approach. One of the four children involved (Tony) had been recognised by the school as having special needs related to learning difficulties and had not been well integrated into group activities. Here, however, he is actively included, his contributions are treated as valuable and the others ensure that he is able to follow and participate in the discussion. They check that he has understood and seek to clarify their suggestions and explanations. Although Tony does not make many verbal contributions to the discussion, he is engaged with the group talk and participates by following the conversation carefully. The initial dilemma – whether the function is 'halving' or 'subtract 2' – is resolved by testing other numbers. During the discussion surrounding this, though their reasoning is imperfect, the group engage with each other's ideas, collaborating to construct and test a hypothesis. Throughout Transcript 5, the children in Group B are careful to ensure that they have reached an agreement before moving on to the next step. Note that Kylie models some of the key Thinking Together phrases that enable the group to structure their discussion and to talk effectively – phrases which have been regularly used and highlighted by their teacher. Although we cannot make a statistical comparison between particular classes (because of the relatively small numbers involved), it was noticeable that the children in Group B's more 'dialogic' class achieved better post-intervention grades in the maths assessment than those in Group A's class.

The interviews with teachers and children in target classes were also used to assess the effects of the intervention programme on learning activity and social interaction. Towards the end of the project, teachers and children in the focal group of each target class were interviewed about their perceptions of its impact on the ways they worked. Teachers also had regular opportunities to report observed changes in speaking and listening to researchers. The responses of the children who undertook the Thinking Together lessons were positive, suggesting that they recognised an improvement in the way they were able to use talk to get things done when they were working in groups. For example:

> 'It has helped us if we are working in groups – now we've got the rules for it as well it's made us think, "Oh, if one person's talking we can't barge in and talk in front of them." . . . We normally take it in our turns and say "What do you think?" instead of leaving someone out. . . . (I'm not) afraid to challenge someone with their answer – (I) don't just sit there and say "All right – pick that one. I don't care". (It) makes us feel more confident if we're in a group.'

Some children's comments refer to the value of Exploratory Talk for helping to assess various alternatives before reaching decisions:

> 'Before the project, if we'd been sitting in the group and got one answer we'd say like – "Oh just say it's that" but now we've been thinking "Oh let's think of another answer it could be as well", not just . . . saying "Oh it looks like that one". Try each.'

Some reported that they learned more by pooling their ideas than by working alone:

> 'It's easier to work in a group than it is on your own because then you've got the time to talk to the person you are working with . . . If you both get the same answer you know it's got to be right because two people have got more chance than just you working it out on your own. . . . Even if you do get it wrong you've got it wrong as a group and not just as a person.'

In interviews, teachers reported similar changes to those mentioned by the children:

> 'They are learning a lot more collaboratively, and listening to each other rather than just hearing each other and they make sure that everyone in the group is involved. They feel more empowered . . . '

> 'The teachers who have (my old class) now have commented on how they are more able to discuss things in groups, not just in their learning but in social aspects as well.'

Discussion and Conclusions

The results reported here provide support for our first main hypothesis: that providing children with guidance and practice in how to use language for reasoning would enable them to use language more effectively as a tool for working on maths problems together. We have demonstrated that the Thinking Together programme enabled children in primary schools to work together more effectively and improve their language and reasoning skills. We also have support for the second hypothesis: that improving the quality of children's use of language for reasoning together would improve their individual learning and understanding of mathematics. This finding is consistent with our earlier reported findings related to the study of science (e.g. Mercer *et al.*, 2004). Our results support claims for the value of collaborative approaches to the learning of mathematics (Sfard & Kieran, 2001; Yackel *et al.*, 1991). We also provide evidence to support the view that the teacher is an important model and guide for pupils' use of language for reasoning.

More generally, our results enhance the validity of a sociocultural theory of education by providing empirical support for the Vygotskian claim that language-based, social interaction (intermental activity) has a developmental influence on individual thinking (intramental activity). More precisely, we have shown how the quality of dialogue between teachers and learners, and amongst learners, is of crucial importance if it is to have a significant influence on learning and educational attainment. By showing that teachers' encouragement of children's use of certain ways of using language leads to better learning and conceptual understanding in maths, we have also provided empirical support for the sociocultural conception of mathematics education as successful induction into a community of practice, as discussed for example by Forman (1996) and Barwell *et al.* (2005). Our findings are also illustrative of the value of Alexander's (2004) concept of 'dialogic teaching', as they show how judgements about the quality of the engagement between teachers and learners can be drawn from an analysis of both the structure and the pragmatic functions of teacher–student discourse. We discussed the extent to which teacher–pupil talk had a monologic or dialogic structure, the kinds of opportunities which children were offered to contribute to discussion, the ways that children's contributions are used by teachers to develop joint consideration of a topic and the role of the teachers as a model for children's own use of language as a tool for thinking.

The Thinking Together intervention programme was carefully designed to include both group-based peer group activities and teacher guidance. The success of its implementation supports the view that the development of mathematical understanding is best assisted by a careful combination of peer group interaction and expert guidance. Our findings indicate that if teachers provide children with an explicit, practical introduction to the use of language for collective reasoning, then children learn better ways of thinking collectively and better ways of thinking alone. However, we have also illustrated some variation in the ways that teachers enacted the dialogic principles underlying the programme as they interacted with their classes – variation which seems to have adversely affected its implementation in some classes. While it would be unreasonable to expect all teachers to give the same commitment to a research study intervention, these findings nevertheless have made us review critically the in-service training about Thinking Together we provide for teachers.

The wider programme of research we have described has already generated materials for the professional development of teachers and the implementation of the Thinking Together approach (Dawes & Sams, 2004; Dawes *et al*., 2003). These set out the structure of the Thinking Together programme, the teaching strategies involved in its implementation and a series of activities in the form of lesson plans. Our findings have also been incorporated into an Open University online in-service course for teachers (Open University, 2004) and are recognised in national educational guidance for teachers (e.g. QCA, 2003a, 2003b). A related project on the use of ICT in mathematics teaching, building on the methods and results described here, has generated new software and teacher guidance for the use of ICT in maths education (Sams *et al*., 2004).

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